$\mathbf{u} = \mathbf{N}\mathbf{d} \qquad \boldsymbol{\varepsilon} = \mathbf{B}\mathbf{d} \qquad \boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} = \mathbf{D}\mathbf{B}\mathbf{d} = \mathbf{S}\mathbf{d}$

$$\mathbf{Q} = \int_{Sq} \mathbf{N}_S^T \mathbf{q}_b dS + \int_V \mathbf{N}^T \mathbf{q} dV + \int_V \mathbf{B}^T \mathbf{D}\boldsymbol{\varepsilon}_0 dV - \int_V \mathbf{B}^T \boldsymbol{\sigma}_0 dV$$

$$\mathbf{d}_a^* = \mathbf{K}_{aa}^{*-1}\mathbf{S}_a^* - \mathbf{K}_{aa}^{*-1}\mathbf{K}_{ap}^*\mathbf{d}_p^* \qquad \mathbf{R}_p^* = \mathbf{S}_p^* - \mathbf{Q}_p^* = \mathbf{K}_{pa}^*\mathbf{d}_a^* + \mathbf{K}_{pp}^*\mathbf{d}_p^* - \mathbf{Q}_p^*$$

LINIJSKI KONAČNI ELEMENTI

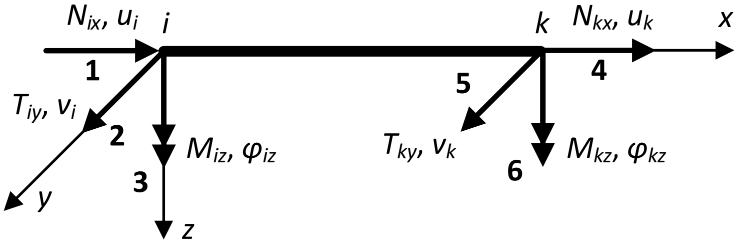
Aksijalno naprezanje

$$N_1 = 1 - \frac{x}{L} \qquad N_4 = \frac{x}{L} \qquad \mathbf{B} = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \qquad \mathbf{D} = [EA]$$

$$\mathbf{k} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix}$$

$$\mathbf{k}^* = \frac{EA}{L}\begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha & -\cos^2\alpha & -\cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha & -\cos\alpha\sin\alpha & -\sin^2\alpha \\ -\cos^2\alpha & -\cos\alpha\sin\alpha & \cos^2\alpha & \cos\alpha\sin\alpha \\ -\cos\alpha\sin\alpha & -\sin^2\alpha & \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$$

Aksijalno naprezanje i savijanje u $x - y$ ravni



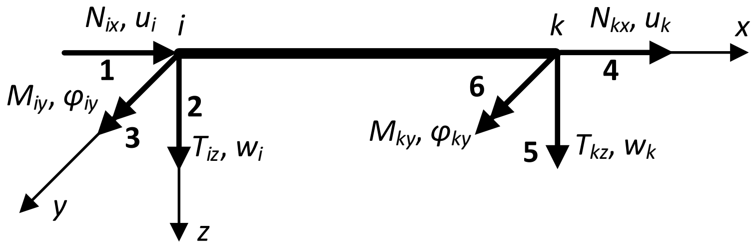
$$N_2 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \qquad N_3(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2} \qquad N_5 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \qquad N_6 = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

$$\mathbf{B}_{xy} = \left[-\frac{6}{L^2} + \frac{12x}{L^3} \quad -\frac{4}{L} + \frac{6x}{L^2} \quad \frac{6}{L^2} - \frac{12x}{L^3} \quad -\frac{2}{L} + \frac{6x}{L^2} \right] \qquad \mathbf{D}_{xy} = \left[EI_z \right]$$

$$\mathbf{k}_{xy} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix}$$

$$T_y = -EI_z \frac{6\left(2v_1 - 2v_2 + L\left(\varphi_{1z} + \varphi_{2z}\right)\right)}{L^3}$$

Aksijalno naprezanje i savijanje u $x - z$ ravni



$$N_2 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \qquad N_3 = -x + \frac{2x^2}{L} - \frac{x^3}{L^2} \qquad N_5 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \qquad N_6 = \frac{x^2}{L} - \frac{x^3}{L^2}$$

$$\mathbf{B}_{xz} = \left[\frac{6}{L^2} - \frac{12x}{L^3} \quad -\frac{4}{L} + \frac{6x}{L^2} \quad -\frac{6}{L^2} + \frac{12x}{L^3} \quad -\frac{2}{L} + \frac{6x}{L^2} \right] \qquad \mathbf{D}_{xz} = \left[EI_y \right]$$

$$\mathbf{k}_{xz} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_y}{L^3} & -\frac{6EI_y}{L^2} & 0 & -\frac{12EI_y}{L^3} & -\frac{6EI_y}{L^2} \\ 0 & -\frac{6EI_y}{L^2} & \frac{4EI_y}{L} & 0 & \frac{6EI_y}{L^2} & \frac{2EI_y}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_y}{L^3} & \frac{6EI_y}{L^2} & 0 & \frac{12EI_y}{L^3} & \frac{6EI_y}{L^2} \\ 0 & -\frac{6EI_y}{L^2} & \frac{2EI_y}{L} & 0 & \frac{6EI_y}{L^2} & \frac{4EI_y}{L} \end{bmatrix}$$

$$T_z = EI_y \frac{6\left(-2w_1 + 2w_2 + L\left(\varphi_{1y} + \varphi_{2y}\right)\right)}{L^3}$$

Matrica transformacije

$$\mathbf{T} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformacija

$$\mathbf{k}^* = \mathbf{T}^T\mathbf{k}\mathbf{T}$$

$\mathbf{d} = \mathbf{T}\mathbf{d}^*$ $\mathbf{Q} = \mathbf{T}\mathbf{Q}^*$ $\mathbf{R} = \mathbf{T}\mathbf{R}^*$

$\mathbf{d}^* = \mathbf{T}^T\mathbf{d}$ $\mathbf{Q}^* = \mathbf{T}^T\mathbf{Q}$ $\mathbf{R}^* = \mathbf{T}^T\mathbf{R}$

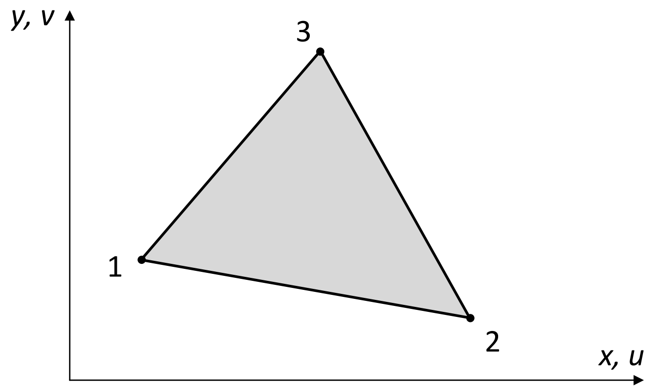
RAVANSKO STANJE NAPONA

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

RAVANSKO STANJE DEFORMACIJE

$$\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

TROUGAONI KONAČNI ELEMENT (CST KE). Linearna interpolacija



$$a_1 = x_2 y_3 - x_3 y_2, \quad a_2 = x_3 y_1 - x_1 y_3, \quad a_3 = x_1 y_2 - x_2 y_1$$
$$b_1 = y_2 - y_3, \quad b_2 = y_3 - y_1, \quad b_3 = y_1 - y_2$$
$$c_1 = x_3 - x_2, \quad c_2 = x_1 - x_3, \quad c_3 = x_2 - x_1$$

$$A = \frac{1}{2}\left(a_1 + a_2 + a_3\right)$$

$$N_i = \frac{1}{2A}\left(a_i + b_i x + c_i y\right), \qquad i = 1,2,3$$

$$\mathbf{B} = \frac{1}{2A}\begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix}$$

$$\mathbf{k} = \frac{h}{4A}\begin{bmatrix} b_1 & 0 & c_1 \\ 0 & c_1 & b_1 \\ b_2 & 0 & c_2 \\ 0 & c_2 & b_2 \\ b_3 & 0 & c_3 \\ 0 & c_3 & b_3 \end{bmatrix} \mathbf{D} \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix}$$

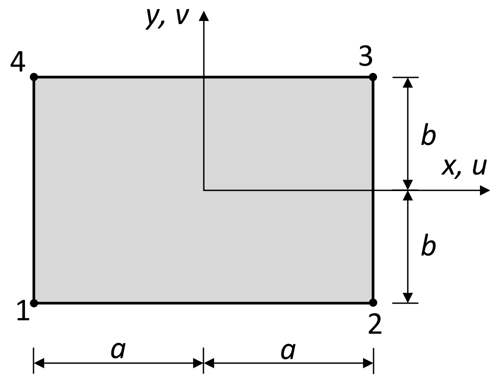
ili

$$\mathrm{k} = \frac{\mathrm{h}}{4\,\mathrm{A}}\left(\begin{array}{ccc} b1^2\,D11 + c1^2\,D33 & b1\,c1\,(D12 + D33) & b1\,b2\,D11 + c1\,c2\,D33 \\ b1\,c1\,(D21 + D33) & c1^2\,D22 + b1^2\,D33 & b2\,c1\,D21 + b1\,c2\,D33 \\ b1\,b2\,D11 + c1\,c2\,D33 & b2\,c1\,D12 + b1\,c2\,D33 & b2^2\,D11 + c2^2\,D33 \\ b1\,c2\,D21 + b2\,c1\,D33 & c1\,c2\,D22 + b1\,b2\,D33 & b2\,c2\,(D21 + D33) \\ b1\,b3\,D11 + c1\,c3\,D33 & b3\,c1\,D12 + b1\,c3\,D33 & b2\,b3\,D11 + c2\,c3\,D33 \\ b1\,c3\,D21 + b3\,c1\,D33 & c1\,c3\,D22 + b1\,b3\,D33 & b2\,c3\,D21 + b3\,c2\,D33 \end{array}\right. \cdots$$

$$\cdots \left.\begin{array}{ccc} b1\,c2\,D12 + b2\,c1\,D33 & b1\,b3\,D11 + c1\,c3\,D33 & b1\,c3\,D12 + b3\,c1\,D33 \\ c1\,c2\,D22 + b1\,b2\,D33 & b3\,c1\,D21 + b1\,c3\,D33 & c1\,c3\,D22 + b1\,b3\,D33 \\ b2\,c2\,(D12 + D33) & b2\,b3\,D11 + c2\,c3\,D33 & b2\,c3\,D12 + b3\,c2\,D33 \\ c2^2\,D22 + b2^2\,D33 & b3\,c2\,D21 + b2\,c3\,D33 & c2\,c3\,D22 + b2\,b3\,D33 \\ b3\,c2\,D12 + b2\,c3\,D33 & b3^2\,D11 + c3^2\,D33 & b3\,c3\,(D12 + D33) \\ c2\,c3\,D22 + b2\,b3\,D33 & b3\,c3\,(D21 + D33) & c3^2\,D22 + b3^2\,D33 \end{array}\right)$$

$$\mathrm{S} = \frac{1}{2\,\mathrm{A}}\begin{pmatrix} b1\,D11 & c1\,D12 & b2\,D11 & c2\,D12 & b3\,D11 & c3\,D12 \\ b1\,D21 & c1\,D22 & b2\,D21 & c2\,D22 & b3\,D21 & c3\,D22 \\ c1\,D33 & b1\,D33 & c2\,D33 & b2\,D33 & c3\,D33 & b3\,D33 \end{pmatrix}$$

PRAVOUGAONI KONAČNI ELEMENT. Bilinearna interpolacija



$$N_1 = \frac{1}{4}\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right) \qquad N_2 = \frac{1}{4}\left(1+\frac{x}{a}\right)\left(1-\frac{y}{b}\right)$$

$$N_3 = \frac{1}{4}\left(1+\frac{x}{a}\right)\left(1+\frac{y}{b}\right) \qquad N_4 = \frac{1}{4}\left(1-\frac{x}{a}\right)\left(1+\frac{y}{b}\right)$$

$$\mathbf{B} = \frac{1}{4ab}\begin{bmatrix} -b+y & 0 & b-y & 0 & b+y & 0 & -b-y & 0 \\ 0 & -a+x & 0 & -a-x & 0 & a+x & 0 & a-x \\ -a+x & -b+y & -a-x & b-y & a+x & b+y & a-x & -b-y \end{bmatrix}$$

$$k = \frac{h}{12\,ab}\left(\begin{matrix} 4\left(b^2\,D11 + a^2\,D33\right) & 3\,a\,b\,(D12 + D33) & -4\,b^2\,D11 + 2\,a^2\,D33 & 3\,a\,b\,(D12 - D33) \\ 3\,a\,b\,(D21 + D33) & 4\left(a^2\,D22 + b^2\,D33\right) & 3\,a\,b\,(-D21 + D33) & 2\,a^2\,D22 - 4\,b^2\,D33 \\ -4\,b^2\,D11 + 2\,a^2\,D33 & 3\,a\,b\,(-D12 + D33) & 4\left(b^2\,D11 + a^2\,D33\right) & -3\,a\,b\,(D12 + D33) \\ 3\,a\,b\,(D21 - D33) & 2\,a^2\,D22 - 4\,b^2\,D33 & -3\,a\,b\,(D21 + D33) & 4\left(a^2\,D22 + b^2\,D33\right) \\ -2\left(b^2\,D11 + a^2\,D33\right) & -3\,a\,b\,(D12 + D33) & 2\,b^2\,D11 - 4\,a^2\,D33 & 3\,a\,b\,(-D12 + D33) \\ -3\,a\,b\,(D21 + D33) & -2\left(a^2\,D22 + b^2\,D33\right) & 3\,a\,b\,(D21 - D33) & -4\,a^2\,D22 + 2\,b^2\,D33 \\ 2\,b^2\,D11 - 4\,a^2\,D33 & 3\,a\,b\,(D12 - D33) & -2\left(b^2\,D11 + a^2\,D33\right) & 3\,a\,b\,(D12 + D33) \\ 3\,a\,b\,(-D21 + D33) & -4\,a^2\,D22 + 2\,b^2\,D33 & 3\,a\,b\,(D21 + D33) & -2\left(a^2\,D22 + b^2\,D33\right) \end{matrix}\right. \cdots$$

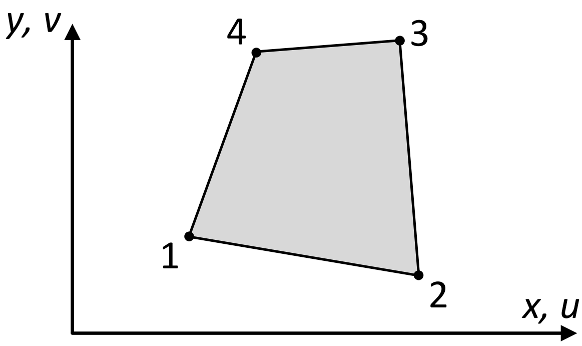
$$\cdots \left.\begin{matrix} -2\left(b^2\,D11 + a^2\,D33\right) & -3\,a\,b\,(D12 + D33) & 2\,b^2\,D11 - 4\,a^2\,D33 & 3\,a\,b\,(-D12 + D33) \\ -3\,a\,b\,(D21 + D33) & -2\left(a^2\,D22 + b^2\,D33\right) & 3\,a\,b\,(D21 - D33) & -4\,a^2\,D22 + 2\,b^2\,D33 \\ 2\,b^2\,D11 - 4\,a^2\,D33 & 3\,a\,b\,(D12 - D33) & -2\left(b^2\,D11 + a^2\,D33\right) & 3\,a\,b\,(D12 + D33) \\ 3\,a\,b\,(-D21 + D33) & -4\,a^2\,D22 + 2\,b^2\,D33 & 3\,a\,b\,(D21 + D33) & -2\left(a^2\,D22 + b^2\,D33\right) \\ 4\left(b^2\,D11 + a^2\,D33\right) & 3\,a\,b\,(D12 + D33) & -4\,b^2\,D11 + 2\,a^2\,D33 & 3\,a\,b\,(D12 - D33) \\ 3\,a\,b\,(D21 + D33) & 4\left(a^2\,D22 + b^2\,D33\right) & 3\,a\,b\,(-D21 + D33) & 2\,a^2\,D22 - 4\,b^2\,D33 \\ -4\,b^2\,D11 + 2\,a^2\,D33 & 3\,a\,b\,(-D12 + D33) & 4\left(b^2\,D11 + a^2\,D33\right) & -3\,a\,b\,(D12 + D33) \\ 3\,a\,b\,(D21 - D33) & 2\,a^2\,D22 - 4\,b^2\,D33 & -3\,a\,b\,(D21 + D33) & 4\left(a^2\,D22 + b^2\,D33\right) \end{matrix}\right)$$

$$S = \frac{1}{4\,ab}\begin{pmatrix} D11\,(-b+y) & D12\,(-a+x) & D11\,(b-y) & -D12\,(a+x) & \dots \\ D21\,(-b+y) & D22\,(-a+x) & D21\,(b-y) & -D22\,(a+x) & \dots \\ D33\,(-a+x) & D33\,(-b+y) & -D33\,(a+x) & D33\,(b-y) & \dots \end{pmatrix}$$

$$\dots \begin{matrix} D11\,(b+y) & D12\,(a+x) & -D11\,(b+y) & D12\,(a-x) \\ D21\,(b+y) & D22\,(a+x) & -D21\,(b+y) & D22\,(a-x) \\ D33\,(a+x) & D33\,(b+y) & D33\,(a-x) & -D33\,(b+y) \end{matrix}\Bigg)$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_1 & & & \\ & \mathbf{T}_2 & & \\ & & \mathbf{T}_3 & \\ & & & \mathbf{T}_4 \end{bmatrix} \qquad \mathbf{T}_i = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \quad i = 1,2,3,4$$

IZOPARAMETARSKI KONAČNI ELEMENT SA 4. ČVORA



$$N_i(\xi,\eta) = \frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta), \qquad i = 1,2,3,4$$

$$\mathbf{B} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix}$$

$$\mathbf{k} = \int_{-1}^{1}\int_{-1}^{1} \mathbf{B}^T\mathbf{D}\mathbf{B}h\det\mathbf{J}d\xi d\eta \qquad \mathbf{Q} = \int_{-1}^{1}\int_{-1}^{1} \mathbf{N}^T\mathbf{q}h\det\mathbf{J}d\xi d\eta$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}x_i & \sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}y_i \\ \sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}x_i & \sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}y_i \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$\frac{\partial N_1}{\partial \xi}=\frac{1}{4}(-1+\eta) \qquad \frac{\partial N_2}{\partial \xi}=\frac{1}{4}(1-\eta) \qquad \frac{\partial N_3}{\partial \xi}=\frac{1}{4}(1+\eta) \qquad \frac{\partial N_4}{\partial \xi}=\frac{1}{4}(-1-\eta)$$

$$\frac{\partial N_1}{\partial \eta}=\frac{1}{4}(-1+\xi) \qquad \frac{\partial N_2}{\partial \eta}=\frac{1}{4}(-1-\xi) \qquad \frac{\partial N_3}{\partial \eta}=\frac{1}{4}(1+\xi) \qquad \frac{\partial N_4}{\partial \eta}=\frac{1}{4}(1-\xi)$$

$$J_{11}=\frac{\partial x}{\partial \xi}=\sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}x_i=\frac{1}{4}\left[x_1(-1+\eta)+x_2(1-\eta)+x_3(1+\eta)+x_4(-1-\eta)\right]$$

$$J_{12}=\frac{\partial y}{\partial \xi}=\sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}y_i=\frac{1}{4}\left[y_1(-1+\eta)+y_2(1-\eta)+y_3(1+\eta)+y_4(-1-\eta)\right]$$

$$J_{21}=\frac{\partial x}{\partial \eta}=\sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}x_i=\frac{1}{4}\left[x_1(-1+\xi)+x_2(-1-\xi)+x_3(1+\xi)+x_4(1-\xi)\right]$$

$$J_{22}=\frac{\partial y}{\partial \eta}=\sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}y_i=\frac{1}{4}\left[y_1(-1+\xi)+y_2(-1-\xi)+y_3(1+\xi)+y_4(1-\xi)\right]$$

$$\mathbf{J}^{-1}=\frac{1}{\det\mathbf{J}}\begin{bmatrix}\sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}y_i & -\sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}y_i\\ -\sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}x_i & \sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}x_i\end{bmatrix}=\frac{1}{\det\mathbf{J}}\begin{bmatrix}J_{22} & -J_{12}\\ -J_{21} & J_{11}\end{bmatrix}$$

$$\det\mathbf{J}=J_{11}J_{22}-J_{12}J_{21}$$

$$\begin{Bmatrix}\frac{\partial N_i}{\partial x}\\ \frac{\partial N_i}{\partial y}\end{Bmatrix}=\mathbf{J}^{-1}\begin{Bmatrix}\frac{\partial N_i}{\partial \xi}\\ \frac{\partial N_i}{\partial \eta}\end{Bmatrix}\Rightarrow \begin{aligned}\frac{\partial N_i}{\partial x}&=\frac{1}{\det\mathbf{J}}\left(J_{22}\frac{\partial N_i}{\partial \xi}-J_{12}\frac{\partial N_i}{\partial \eta}\right)\\ \frac{\partial N_i}{\partial y}&=\frac{1}{\det\mathbf{J}}\left(-J_{21}\frac{\partial N_i}{\partial \xi}+J_{12}\frac{\partial N_i}{\partial \eta}\right)\end{aligned}$$

$$\frac{\partial N_1}{\partial x}=\frac{1}{8\det\mathbf{J}}\left[y_2(1-\eta)+y_3(-\xi+\eta)+y_4(-1+\xi)\right] \quad \frac{\partial N_1}{\partial y}=\frac{1}{8\det\mathbf{J}}\left[x_2(-1+\eta)+x_3(\xi-\eta)+x_4(1-\xi)\right]$$

$$\frac{\partial N_2}{\partial x}=\frac{1}{8\det\mathbf{J}}\left[y_1(-1+\eta)+y_3(1+\xi)+y_4(-\xi-\eta)\right] \quad \frac{\partial N_2}{\partial y}=\frac{1}{8\det\mathbf{J}}\left[x_1(1-\eta)+x_3(-1-\xi)+x_4(\xi+\eta)\right]$$

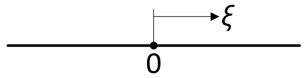
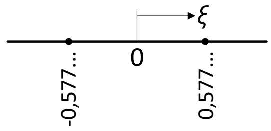
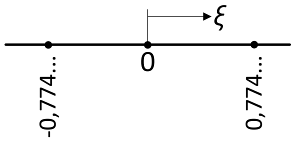
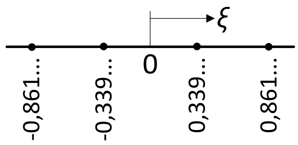
$$\frac{\partial N_3}{\partial x}=\frac{1}{8\det\mathbf{J}}\left[y_1(\xi-\eta)+y_2(-1-\xi)+y_4(1+\eta)\right] \quad \frac{\partial N_3}{\partial y}=\frac{1}{8\det\mathbf{J}}\left[x_1(-\xi+\eta)+x_2(1+\xi)+x_4(-1-\eta)\right]$$

$$\frac{\partial N_4}{\partial x}=\frac{1}{8\det\mathbf{J}}\left[y_1(1-\xi)+y_2(\xi+\eta)+y_3(-1-\eta)\right] \quad \frac{\partial N_4}{\partial y}=\frac{1}{8\det\mathbf{J}}\left[x_1(-1+\xi)+x_2(-\xi-\eta)+x_3(1+\eta)\right]$$

NUMERIČKA INTEGRACIJA

$$\int_{-1}^{1}\int_{-1}^{1} F(\xi,\eta)\,d\xi\, d\eta = \sum_{j=1}^{nj} w_j\left(\sum_{i=1}^{ni} w_i F(\xi_i,\eta_j)\right) = \sum_{i=1}^{ni}\sum_{j=1}^{nj} w_{ij} F(\xi_i,\eta_j), \qquad w_{ij} = w_i w_j$$

Tačke integracije i težinski koeficijenti

n	Stepen polinoma	i	ξ_i	w_i	
1	1	1	0,0	2,0	
2	3	1 2	$-1/\sqrt{3}$ $1/\sqrt{3}$	1,0 1,0	
3	5	1 2 3	$-\sqrt{3/5}$ 0 $\sqrt{3/5}$	5/9 8/9 5/9	
4	7	1 2 3 4	−0,8611363116 −0,3399810436 0,3399810436 0,8611363116	0,3478548451 0,6521451549 0,6521451549 0,3478548451	
5	9	1 2 3 4 5	−0,9061798459 −0,5384693101 0 0,5384693101 0,9061798459	0,2369268851 0,4786286705 0,5688888889 0,4786286705 0,2369268851